

On the AdS/CFT Dual of Deconstruction

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Abstract

We consider a class of non-supersymmetric gauge theories obtained by orbifolding the $N = 4$ super-Yang-Mills theories. We focus on the resulting quiver theories in their deconstructed phase, both at small and large coupling, where a fifth dimension opens up. In particular we investigate the rôle played by this extra dimension when evaluating the rectangular Wilson loops encoding the interaction potential between quarks located at different points in the orbifold. The large coupling potential of the deconstructed quiver theory is determined using the AdS/CFT correspondence and analysing the corresponding minimal surface solution for the dual gravitational metric. At small coupling, the potential between quarks decreases with their angular distance while at strong coupling we find a linear dependence at large distance along the (deconstructed) fifth dimension.

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1 Introduction

Recently there has been a renewed interest in extensions of the standard model which differ from the supersymmetric framework. One of the initial motivations for the supersymmetric extensions of the standard model is the possibility of preserving a hierarchy between the weak scale and the unification scale. In supersymmetric theories this springs from the delicate balance between bosonic and fermionic contributions to radiative corrections. In particular at the one loop level the quadratic divergences exactly vanish. In the context of softly broken supergravity, the supertrace of the square mass matrix is proportional to the gravitino mass, hence field independent and preserving the high energy features of unbroken supersymmetry. The deconstructed models [1, 2, 3, 4] offer an alternative to this scenario. In these models the quadratic divergences are absent due to the equivalence with a fifth dimensional gauge theory forbidding the appearance of mass terms.

The deconstructed models have a structure highly reminiscent of quiver theories [5]. Indeed by considering the field theory limit of $D3$ brane configurations in the vicinity of an orbifold singularity one can construct non-supersymmetric gauge theories with a product gauge group $U(n)^\Gamma$ and fields in bifundamental representations. These theories have proved to be useful in building string realizations of the standard model [6]. Though non-supersymmetric, the quiver theories share another feature with deconstructed models. Indeed the quadratic divergences vanish exactly [7]. In the deconstructed phase of quiver theories, where the gauge group is broken to the diagonal gauge group $U(n)^\Gamma \rightarrow U(n)_D$, this springs from an underlying custodial supersymmetry. Similarly to the deconstructed models, the quiver theories in the deconstructed phase are equivalent to a fifth dimensional $U(n)$ gauge theory in the large Γ limit.

In this paper we will investigate the properties of (rectangular) Wilson loops for quiver theories in the deconstructed phase. In particular we will focus on the interaction potential between *twisted* quarks, i.e. quarks corresponding to open strings with end-points in different sectors of the orbifold cover. In section 2, we recall some ingredients about quiver theories. In section 3, we compute the quark potential at weak coupling. The development of a fifth dimension is made explicit in the R_5/L^2 behaviour of the potential. The L^2 dependence signals the propagation of massless degrees of freedom in five dimensions while the R_5 factor is the only dimension-full constant of the five dimensional theory. In sections 4, 5 and 6 we analyse the strong coupling behaviour. In section 4, we formulate the computation of the rectangular Wilson loop in terms of a minimal surface having the loop as boundary (“Wilson surface”) using the AdS/CFT correspondence. In section 5, we analyze the geometry of the Wilson surface in and outside the deconstructed region. In section 6, we finally compute the potential and thus the force between quarks along the deconstructed dimension. We find that the quarks have a linear potential at large (angular) distance, a property reminiscent of confinement along the fifth dimension.

2 Deconstructing Non Supersymmetric Quivers

We are interested in certain non-supersymmetric gauge theories whose structure can be inferred from the world-volume dynamics of $D3$ branes in the neighbourhood of an orbifold singularity. The breaking of supersymmetry is due to the orbifolding which does not preserve the original $N = 4$ invariance of the low energy dynamics on $D3$ branes.

Consider the type IIB string theory with a stack of n coinciding D3 branes. It is well known that the gauge bosons and fermions living on the worldvolume of the D3 branes form a 4d $N = 4$ supersymmetric Yang-Mills model with gauge group $U(n)$. The six transverse dimensions represent, from the point of view of the 4d theory living on the worldvolume, six extra nongravitational dimensions. The spectrum and interactions of that model are the same as the ones obtained by dimensional reduction of the $N = 1$ $U(n\Gamma)$ gauge theory living in 10 dimensions.

One can obtain a theory with fewer supersymmetries than $N = 4$ by dividing the extra dimensions by a discrete group Z_Γ and embedding this orbifold group into the gauge group $U(n\Gamma)$. The spectrum consists of the fields which are invariant under the combined geometric and gauge actions of Z_Γ (see the quiver diagram in Fig.1).

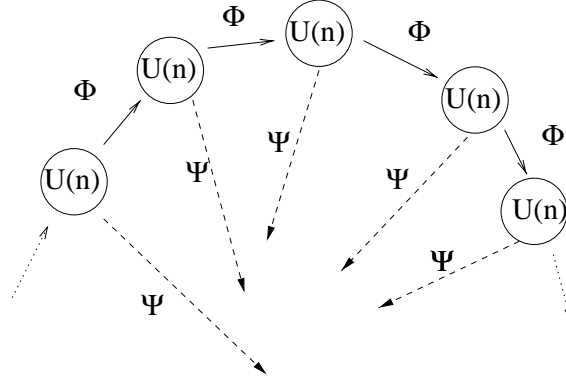


Figure 1: *Typical quiver diagram.* The bosonic (Φ) and fermionic (Ψ) fields corresponding to zero-modes of open strings with ends on branes are schematically represented by arrows. The figure corresponds to the non-supersymmetric case $\tilde{a}_1 \equiv 2a_4 \equiv 2a_1$.

The field interaction terms are consistently truncated to yield a smaller daughter gauge theory. The truncation process breaks the gauge group and some (or all) supersymmetries. The gauge symmetry breaking is dictated by the embedding of the generator of Z_Γ into $U(n\Gamma)$. The matrix γ that represents the gauge action of Z_Γ is chosen to be of the form of a direct sum of Γ unit matrices of dimensions $n \times n$, each multiplied respectively by ω^i with $\omega = e^{\frac{2\pi}{\Gamma}i}$. The invariant components of the gauge fields fulfill the condition

$$A = \gamma A \gamma^{-1} \quad (1)$$

where A is a matrix in the adjoint representation of $U(n\Gamma)$. This leaves invariant the subgroup $U(n)^\Gamma$.

There are four generations of Weyl fermions whose invariant components must obey the condition

$$\psi^i = \omega^{a_i} \gamma \psi^i \gamma^{-1} \quad (2)$$

where $i = 1, \dots, 4$ and

$$a_1 + a_2 + a_3 + a_4 \equiv 0 \pmod{\Gamma} . \quad (3)$$

The invariant fermions transform in the bifundamental representations $(\mathbf{n}_l, \bar{\mathbf{n}}_{l+a_i})$ of the broken gauge group where l numbers blocks of the original $n\Gamma \times n\Gamma$ matrices. Furthermore, one obtains three generations of complex bosons ϕ^i , $i = 1, 2, 3$, whose invariant components fulfill the condition

$$\phi^i = \omega^{\tilde{a}_i} \gamma \phi^i \gamma^{-1}. \quad (4)$$

The invariant scalars transform as $(\mathbf{n}_l, \bar{\mathbf{n}}_{l+\tilde{a}_i})$ under the broken gauge group. The integers \tilde{a}_i correspond to the orbifold action $z_i \rightarrow e^{2\pi i \tilde{a}_i / \Gamma} z_i$ on the three complex planes.

The truncated fields have a block structure in the $U(n\Gamma)$ mother gauge group

$$\phi_{lp}^i = \phi_l^i \delta_{p,l+\tilde{a}_i}, \quad \psi_{lp}^i = \psi_l^i \delta_{p,l+a_i}. \quad (5)$$

Supersymmetry is preserved when the group Z_Γ is embedded in $SU(3)$

$$\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 \equiv 0 \pmod{\Gamma}. \quad (6)$$

In that case $a_4 \equiv 0$ and at least one of the fermions can be paired with the gauge bosons, i.e. becoming a gaugino of $N = 1$ supersymmetry. We focus on the non-supersymmetric case $a_4 \neq 0$.

Let us move a stack of n $D3$ branes from the origin. Moving the stacks of n $D3$ branes from the origin corresponds to a diagonal *vev* for each ϕ_l^i when $\tilde{a}_i \neq 0$. This is equivalent to shifting a stack of n branes from the origin by a distance R . Due to the Z_Γ action, the stacks have Γ copies around the fixed point. The gauge group is broken to the diagonal subgroup $U(n)_D$. This is the deconstructed phase of the quiver theory with a breaking pattern $U(n)^\Gamma \rightarrow U(n)_D$.

We are interested in the geometry of the orbifold close to the branes. It is convenient [8, 9] to parametrize the orbifold with the coordinates

$$z_i = r_i e^{i(\alpha_i \theta_1 + \beta_i \theta_2 + \frac{a_i}{\Gamma} \phi)} \quad (7)$$

where the vectors α_i , β_i and a_i are orthogonal and we normalize $\alpha^2 = \beta^2 = 1$. The flat metric on the orbifold reads

$$ds^2 = \sum_{i=1}^3 dz_i d\bar{z}_i = \sum_{i=1}^3 dr_i^2 + R^2 d\theta_1^2 + R^2 d\theta_2^2 + \frac{a^2 R^2}{\Gamma^2} d\phi^2 \quad (8)$$

where $r_i = R(1 + x_i)$ and $x_i \ll 1$. We have defined $a^2 = \tilde{a}_1^2 + \tilde{a}_2^2 + \tilde{a}_3^2$ and assumed that the orbifold acts non-trivially on the three complex planes. One recognizes a circle S^1 parametrized by $\phi \in [0, 2\pi]$ corresponding to the Z_Γ orbit of radius

$$R_{S^1} = \frac{l_s^2}{R_5}, \quad R_5 = \frac{\Gamma l_s^2}{aR}. \quad (9)$$

In the large Γ limit, the vicinity of a stack of n branes corresponds to a cylinder of very small radius R_{S^1} .

In this geometry the stack of $D3$ branes become localized at a point on the circle S^1 . It is identified with its multiple images under the 2π rotation around the S^1 circle. It is now easy to analyse the field theory on the stack of $D3$ branes. Indeed the six dimensions of the orbifold have been replaced by a product $R^3 \times (S_R)^2 \times S^1$ where S_R is a circle of radius R . Now the field theory of a stack of n $D3$ branes localized in $R^3 \times (S_R)^2 \times S^1$ is a $N = 4$ $U(n)$ SYM gauge theory corresponding to the massless open strings joining the stack to itself, see Fig.1. The massive states of the theory are obtained by wrapping open strings around S^1 . The mass spectrum is given by

$$m_k = \frac{2\pi k}{R_5} \quad (10)$$

which is a Kaluza-Klein spectrum of a fifth dimensional theory compactified on a radius R_5 . The appearance of this extra dimension can be understood using an appropriate T -duality [9].

Notice that $R_{S^1} \ll l_s$ as soon as $R_5 \gg l_s$, i.e. as $\Gamma \rightarrow \infty$ keeping R fixed. In the large Γ limit, substringy distances are probed by the D3 branes. It is more appropriate to T-dualize along S^1 . The radius of the new circle becomes the large radius R_5 . Similarly the D3 branes become wrapped D4 branes. On the D4-branes the gauge theory is a $U(n)$ five dimensional gauge theory compactified on a circle of radius R_5 . This is the generalization to quiver theories of the deconstructed models.

In the following we will consider the deconstructed quiver theories both at small and large coupling constant. In particular we will focus on the Wilson lines. At weak coupling the Wilson lines can be understood by moving one of the branes of a stack to infinity and identifying the open string state connecting the stack of n D3 branes to the brane at infinity as a static quark. Such an open string can wind w times around the orbifold. This is easily pictured on the orbifold covering space where the string connects D3 branes belonging to different sectors. We will refer to this situation as *twisted* quarks.

At strong coupling we will obtain relevant information from the supergravity solution generated by the stacks of D3 branes. It has to be noticed that the supergravity background with branes displaced to various points of the circle of radius R has a constant dilaton. Hence the gauge theory coupling constant does not run with the energy scale (which is related by the UV/IR principle to the radial distance scale in the bulk geometry). When we pass to the orbifold the same is obviously true.

However one has to keep in mind that there is a tachyonic twisted mode at the centre of the orbifold which leads to an instability [10]. The behaviour of the dilaton may become non-trivial leading to the fact that a running coupling may be generated at this stage. For the part of the geometry relevant for the calculation of the static potential between static quarks along the deconstructed fifth dimension, and for $R \gg \sqrt{\alpha'}$, the static potential should not be affected by this tachyonic instability if the change of geometry is confined to a finite region, close to the centre of the orbifold [10]. A more detailed study of this instability is out of the scope of the present paper, but certainly deserves further investigation.

3 Rectangular Wilson Loop at Small Coupling

The appearance of a perturbative extra-dimension in the large Γ regime can be investigated by computing the quark-quark potential in the static approximation [11]. It is expected that a $1/L^2$ dependence of the potential in the inter-quark distance L should appear as required by a five-dimensional interpretation. We will show that this is indeed the case. Moreover we will be interested in the potential between quarks belonging to different twisted sectors of the orbifold. Indeed in the universal cover of the orbifold one can place quarks in different sectors identified under the orbifold action. We will compute the potential as a function of the angle between the quarks in the universal cover. In particular we focus on an orbifold action on only one plane, i.e.

$$\tilde{a}_1 \equiv 2a_4, \tilde{a}_2 \equiv 0, \tilde{a}_3 \equiv 0 \text{ mod}(\Gamma) \quad (11)$$

corresponding to

$$a_1 \equiv a_4, a_2 \equiv -a_4, a_3 \equiv -a_4 \text{ mod}(\Gamma) . \quad (12)$$

In the following we will focus on the simplest situation $\tilde{a}_1 \equiv 1 \text{ mod}(\Gamma)$, sketched in Fig.1.

In this section we will compute the small coupling Wilson loop for deconstructed quiver theories [12]

$$W = \frac{1}{\Gamma n} \text{Tr} P e^{ig \int_C (A_m \dot{x}^m - X_i u^i |\dot{x}|)} \quad (13)$$

where C is the Wilson loop contour parametrized by x^m , and u^i is a unit vector representing the coupling to the six real scalar fields X_i , $i = 1 \dots 6$ spanning the six extra dimensions. We will concentrate on the case where the contour is associated to a rotation along one plane in the six extra-dimensional directions. The twist of this contour is parametrized by an angle $\Delta\theta$ representing a rotation between two sectors of the orbifold, see Fig.2.

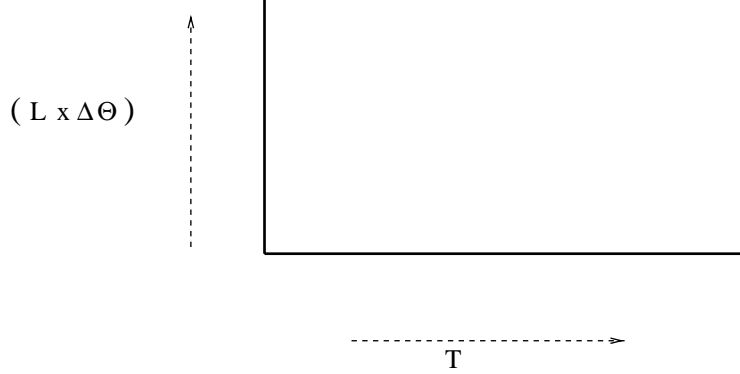


Figure 2: *Rectangular Wilson loop with “twist”*. The space evolution of the fields along the Wilson line is accompanied by a corresponding rotation (“twist”) between sectors of the orbifold, see text.

In our case

$$\vec{u} = (\cos \Delta\theta \frac{\sigma}{L}, \sin \Delta\theta \frac{\sigma}{L}) \quad (14)$$

in between the two quarks as σ runs between 0 and L while

$$\vec{u} = (1, 0) \quad (15)$$

for the static quark sitting at the origin and

$$\vec{u} = (\cos \Delta\theta, \sin \Delta\theta) \quad (16)$$

for the static quark at distance L . The Wilson loop contour is represented in Fig.2.

We also denote by T the time length of the Wilson loop. We will take it to be very large eventually. The Wilson loop can be computed by going to Euclidean space and expanding the connected Green functions to second order in the gauge coupling constant

$$\begin{aligned} \ln \langle W \rangle &= g^2 T \int_C (\langle A_0(0, 0) A_0(L, \tau) \rangle + \langle \phi(0, 0) \bar{\phi}(L, \tau) \rangle e^{i\Delta\theta} \\ &\quad + \langle \phi(0, \tau) \bar{\phi}(L, 0) \rangle e^{-i\Delta\theta}) \end{aligned} \quad (17)$$

where the first argument of $A_0(0, \tau)$ is at the origin of space-time where one quark sits and the second is the time τ parametrizing the time evolution of the quark. We have denoted

$\phi = \frac{X_1 + iX_2}{2}$. In the deconstructed phase one can decompose the fields into massless and massive modes according to [7]

$$A_p = \sqrt{\frac{2}{\Gamma}} \left\{ \sum_{n=0}^{(\Gamma-1)/2} \eta_n \cos\left(\frac{2\pi n}{\Gamma} p\right) A_p^{(n)} + \sum_{n=0}^{(\Gamma-1)/2} \sin\left(\frac{2\pi n}{\Gamma} p\right) \tilde{A}_p^{(n)} \right\} \quad (18)$$

where each field A_p represents one of the Γ blocks and the index (n) numbers the different fields. Here we have identified $\eta_0 = 1/\sqrt{2}$ and $\eta_n = 1$, $n \neq 0$. The fields $A_p^{(n)}$ have masses m_n while the fields $\tilde{A}_p^{(n)}$ have masses $m_{\{\Gamma-n\}}$. Similar expressions hold for the scalar fields with the same mass spectrum. This fact is due to the underlying custodial supersymmetry of the non-supersymmetric quiver theories [7]. Using the propagators

$$\langle A_p^{(n)}(0,0) A_p^{(n)}(L,\tau) \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip_0\tau + i\vec{p}\cdot\vec{L}}}{p_0^2 + p^2 + m_n^2} \quad (19)$$

we find that the Wilson loop can be expressed as

$$\ln \langle W \rangle = g^2 n T (1 + \cos(\Delta\theta)) \int_0^T d\tau G_5 \quad (20)$$

where

$$G_5 = \sum_{n=0}^{\Gamma-1} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip_0\tau + i\vec{p}\cdot\vec{L}}}{p_0^2 + p^2 + m_n^2} \quad (21)$$

and we have removed the self energy contributions. In the deconstruction regime this leads to

$$G_5 = R_5 \int \frac{d^5 p}{(2\pi)^5} \frac{e^{-ip_0\tau + i\vec{p}\cdot\vec{L}}}{p_0^2 + p^2 + p_5^2} \quad (22)$$

which is nothing but the five dimensional propagator

$$G_5 = \frac{R_5}{(\tau^2 + L^2)^{3/2}} \quad (23)$$

For large T we deduce the interaction potential

$$V = \frac{g^2 n R_5}{L^2} \int_0^\infty \frac{1 + \cos \Delta\theta}{(1 + u^2)^{3/2}} = \frac{g^2 n R_5}{L^2} \{1 + \cos \Delta\theta\}. \quad (24)$$

Notice the expected $1/L^2$ behaviour, characteristic of the opening of the fifth dimension at weak coupling.

The potential is periodic in $\Delta\theta$ in the orbifold cover. Antipodal points are such that there is maximal screening with a vanishing interaction potential. As $\Delta\theta$ increases, the potential decreases and the interaction force between two twisted sectors decreases. Finally the potential is proportional to the radius of the compactified fifth dimensional gauge theory R_5 , which sets its dynamical scale.

4 The rectangular Wilson Loop at Strong Coupling

4.1 Dual metric deconstruction

We have seen that in the large Γ limit, deconstructed quiver theories become similar to a fifth dimensional gauge theories. We will now analyse the models in the large gauge coupling limit using the AdS/CFT correspondence [13, 14]. To simplify the analytic treatment we shall concentrate on the case where the orbifold group acts on a single complex plane. The metric due to the presence of the displaced branes is given by

$$ds_{10}^2 = H^{-1/2} ds_4^2 + H^{1/2} ds_6^2 \quad (25)$$

where

$$H = 1 + \sum_{j=1}^{\Gamma} \frac{r_0^4}{|r - r_j|^4} \quad (26)$$

and $r_j = (Re^{2\pi i j/\Gamma}, 0, 0)$ is the location of the j -th image of the displaced brane. The complement metric ds_6^2 is defined up to the orbifold identifications. In the case where the orbifold acts on a single plane we denote by θ the polar angle in that plane. We are interested in computing the Wilson line between quarks belonging to different sectors. In the orbifold cover this amounts to putting quarks in sectors separated by an angle which is a multiple of $2\pi/\Gamma$. At strong coupling the relevant regime is the blown-up vicinity of the displaced brane where

$$H \approx \sum_{i=1}^{\Gamma} \frac{r_0^4}{|r - r_i|^4} \quad (27)$$

while the quarks lie at infinity.

There are various relevant regions. Far away from the branes, space-time becomes isometric to $AdS_5 \times S^5/Z_{\Gamma}$. The quarks are on the boundary of AdS_5 . In the interior of the six extra dimensions, the geometry departs from the AdS_5 behaviour. In particular there is a ring around the stacks of branes where the harmonic function signals the presence of a fifth dimension. The geometrical setting in the orbifold plane has been sketched in Fig.3.

The strong coupling calculation of Wilson loop expectation values reduces, in the classical approximation of the AdS/CFT correspondence, to the evaluation [14, 15, 16] of the bulk minimal surface area bounded by the Wilson loop contour. In this context, the (massive) quarks in the static limit are represented by an open string with ends near infinity in r .

The nature of the minimal surface depends crucially on the harmonic function H and on the physical length of the wilson loop L . In the following we will consider the region

$$x \gg \frac{1}{\Gamma} \quad (28)$$

where $x = r/R - 1$. In that region the harmonic function (26) can be well approximated [11] by

$$H = \Gamma r_0^4 \frac{r^2 + R^2}{(r^2 - R^2)^3} \quad (29)$$

with no angular dependence, i.e. leading to the existence of two conserved quantities \mathbf{E} and \mathbf{L} . Close to the circle containing all the branes, the harmonic function reads

$$H = \frac{\Gamma r_0^4}{4R^4} \frac{1}{x^3} \quad (30)$$

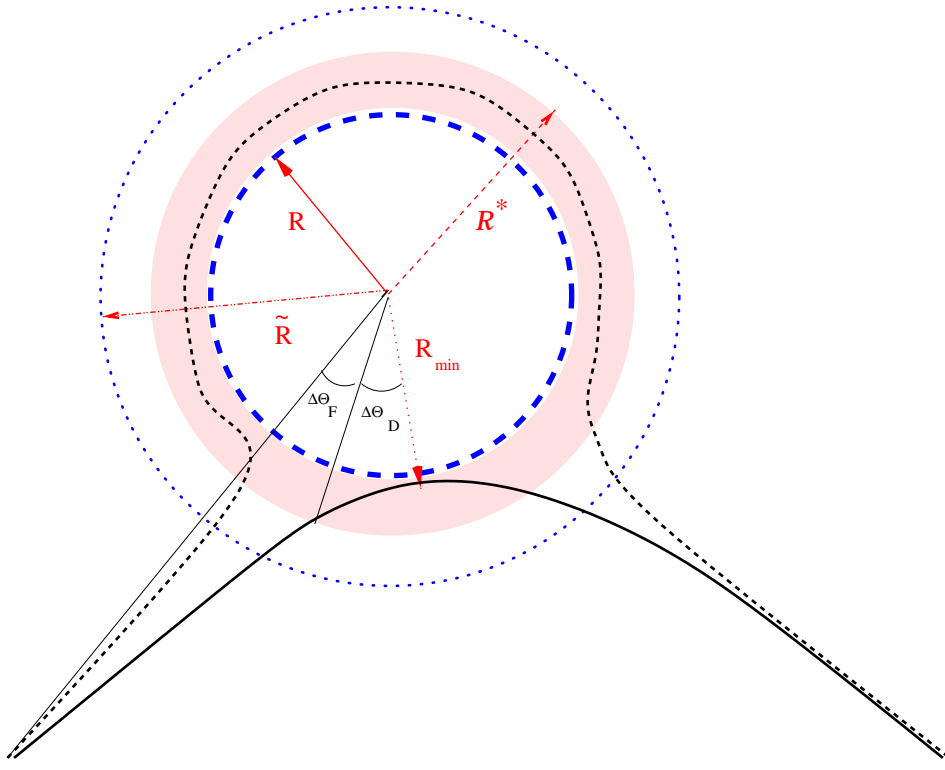


Figure 3: *Wilson surface projected on the “fifth dimension”*. The minimal surface contours are displayed as seen in the orbifold plane. The full (*resp.* dashed) curve correspond to the dominant (*resp.* first subdominant) contribution to the potential (see subsection 4.2).

- R : Location radius of the D_3 -branes : *Thick dashed circle*
- $R \leq r \leq R^*$: Deconstructed phase region : *Hatched ring*
- $r = \tilde{R}$: Lower limit of (approximate) validity of the $AdS \times S_5$ metrics : *Dotted circle*
- $\Delta\Theta_D$ (*resp.* $\Delta\Theta_F$): half the angle spanned by the Wilson contour inside (*resp.* outside) the deconstruction region (see section 5).

which is valid for $1/\Gamma \ll x \leq x_*$ where $x_* < \mathcal{O}(1)$ defines a scale limiting the validity of the approximation. The circle of radius $R^* = R(1 + x_*)$ is the outer edge of the region where the behaviour of the harmonic function is similar to a five dimensional harmonic function decaying with the third power of the distance. We will call the region between R and R^* the “deconstruction domain”.

Now, at very large distances the harmonic function becomes the AdS_5 harmonic function

$$H = \frac{\Gamma r_0^4}{r^4} . \quad (31)$$

For the sake of clarity in the following calculation and discussion, let us denote by $r \equiv \tilde{R}$ the circle beyond which (31) is valid with a sufficiently good approximation. This will in particular contain the region where the quark sources stand. Between these two regions, namely for $R^* < r < \tilde{R}$, the harmonic function H of (29) interpolates smoothly.

4.2 Integrals of motion on the Wilson contour

We are interested in computing the interaction potential, *i.e.* the rectangular Wilson loop

in the large T limit, when quarks lie in different sectors of the orbifold. Within the AdS/CFT correspondence scheme the potential is determined by the minimal surface area swept by a string connecting the boundary quarks [15]. The geometry of this Wilson surface, in particular its behaviour in the deconstructed domain, will determine the basic features of the force between quarks.

By comparison with the original evaluations of Wilson contours [15], there exist some differences that we have to face. The quark sources, represented by D-branes at infinity are *a priori* outside the deconstruction region, see Fig.3. One thus has to take care of how and where the minimal Wilson surface is attracted near the set of orbifolded D-branes. This does require to determine (at least with sufficient accuracy) the Wilson line contour inside and outside the deconstruction region. let us thus first write the general equations determining the minimal surface area.

This area is determined by the surface element

$$dS_2 = (H^{-1} + r'^2 + r^2\theta'^2)^{1/2} d\sigma d\tau . \quad (32)$$

where $' \equiv d/d\sigma$. This leads to the potential

$$V(L, \Delta\theta) = \frac{1}{2\pi\alpha'} \int d\sigma (H^{-1} + r'^2 + r^2\theta'^2)^{1/2} . \quad (33)$$

Let us note that for each fixed $\Delta\theta$ we have at least two series of strings which contribute to the Wilson loop — one which stretches ‘anti-clockwise’ with $\theta_f - \theta_i = \Delta\theta + 2\pi n$, and one which runs clockwise with $\theta_f - \theta_i = 2n\pi - \Delta\theta$. Therefore the Wilson loop expectation value is given by the infinite sum

$$\langle W(T \times L) \rangle = \sum_{n \geq 0} (e^{-TV(L, \Delta\theta + 2\pi n)} + e^{-TV(L, 2\pi(n+1) - \Delta\theta)}) . \quad (34)$$

For $T \rightarrow \infty$ only the two first contributions will survive giving the effective potential

$$V_{eff}(L, \Delta\theta) = \min \{V(L, \Delta\theta), V(L, 2\pi - \Delta\theta)\} \quad (35)$$

The potential is explicitly periodic. In the following we will just determine the function $V(L, \Delta\theta)$.

The potential can be cast into the form

$$V = \frac{1}{2\pi\alpha'} \int dr (1 + H^{-1}\dot{\sigma}^2 + r^2\dot{\theta}^2)^{1/2} , \quad (36)$$

where $\cdot \equiv d/dr$. In the following we shall use the Lagrangian

$$\mathcal{L} = (1 + H^{-1}\dot{\sigma}^2 + r^2\dot{\theta}^2)^{1/2} . \quad (37)$$

where H does not depend on either σ or θ . This leads to the existence of two integrals of motion analogous to the energy and the angular momentum

$$\begin{aligned} \mathbf{E} &= \frac{H^{-1}\dot{\sigma}}{\mathcal{L}} = cst. \\ \mathbf{l} &= \frac{r^2\dot{\theta}}{\mathcal{L}} = cst. \end{aligned} \quad (38)$$

This implies that the Lagrangian can be expressed as

$$\mathcal{L} = (1 - H\mathbf{E}^2 - \frac{\mathbf{l}^2}{r^2})^{-1/2} \quad (39)$$

The equations of motion can then be deduced

$$\begin{aligned} \dot{\sigma} &= \frac{\mathbf{E}H}{(1 - H\mathbf{E}^2 - \frac{\mathbf{l}^2}{r^2})^{1/2}} \\ \dot{\theta} &= \frac{1}{r^2} \frac{1}{(1 - H\mathbf{E}^2 - \frac{\mathbf{l}^2}{r^2})^{1/2}} \end{aligned} \quad (40)$$

The total length of the Wilson line will be $2L$ where

$$L = \int_{R_{min}}^{\infty} \frac{\mathbf{E}H}{(1 - H\mathbf{E}^2 - \frac{\mathbf{l}^2}{r^2})^{1/2}} dr \quad (41)$$

and the twist angle, $2\Delta\theta$, with

$$\Delta\theta = \int_{R_{min}}^{\infty} \frac{1}{r^2} \frac{1}{(1 - H\mathbf{E}^2 - \frac{\mathbf{l}^2}{r^2})^{1/2}} dr . \quad (42)$$

It is important to note that the minimal surface will not reach the circle of branes but stops at a minimal value R_{min} where the Lagrangian \mathcal{L} , see (39) goes to infinity. More precisely, since then $dr/d\sigma \equiv 1/\dot{\sigma} \propto \mathcal{L}$, the minimized Wilson contour will follow a trajectory near R_{min} , spanning an angle $2\Delta\theta_D$ inside the deconstruction region, and $2\Delta\theta_F$ outside this region, see Fig.3.

5 Dual geometry of deconstruction

5.1 Length and twist of the Wilson line in the deconstruction domain

Let us first focus on the contribution to the length L_D of the Wilson line in the deconstruction region.

Considering (41), together with the harmonic function (30), and using an appropriate rescaling of variable, one obtains

$$L_D = \frac{1}{x_{min}^{1/2}} \frac{\sqrt{\Gamma} r_0^2}{R} \int_1^{x_*/x_{min}} \frac{du}{u^{3/2} \sqrt{u^3 - 1}} \quad (43)$$

where we require that $r = R(1 + x)$ lies within the deconstruction region. One gets

$$x_{min} = \frac{\Gamma r_0^4}{L_D^2 R^2} F^2 \left(\frac{x_*}{x_{min}} \right) \quad (44)$$

where

$$F(y) = \int_1^y \frac{du}{u^{3/2} \sqrt{u^3 - 1}} \equiv \frac{2}{3} (y^3 - 1)^{\frac{1}{2}} y^{\frac{7}{2}} {}_2F_1(7/6, 1; 3/2 | 1 - y^{-3}) . \quad (45)$$

Looking for a solution where $x_{min}/x_* << 1$ for R_{min} to lie in the deconstruction region, we find to leading order

$$x_{min} = \frac{\Gamma r_0^4}{L_D^2 R^2} F^2 \quad (46)$$

where

$$F \equiv F(\infty) = \frac{2\sqrt{\pi}\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})} \approx .86237 . \quad (47)$$

Imposing that $1/\Gamma << x_{min} << 1$ implies that the deconstruction regime is characterized by

$$\frac{F(g^2 n)^{1/2} R_5}{\sqrt{\Gamma}} << L_D << F(g^2 n)^{1/2} R_5 . \quad (48)$$

Note that (48) has the meaning of a limitation of the 4-dimensional distance L_D over which the potential calculation can be done within our approximation scheme. Larger 4D distances will be briefly mentioned in the conclusion.

We find that the integrals of motion (38) can be expressed as

$$\begin{aligned} \mathbf{E} &= \frac{2R^2}{\sqrt{\Gamma} r_0^2} x_{min}^{5/2} \frac{1}{(x_{min}^2 + \frac{\Delta\theta_D^2}{G^2})^{1/2}} \\ \mathbf{l} &= \frac{\Delta\theta_D}{G} \frac{R}{(x_{min}^2 + \frac{\Delta\theta_D^2}{G^2})^{1/2}} . \end{aligned} \quad (49)$$

Hence the twist in the deconstruction region, which we choose as the starting value for our evaluation can be identified with

$$\Delta\theta_D = \int_{x_{min}}^{x_*} \dot{\theta} dx . \quad (50)$$

This leads to

$$\begin{aligned} \mathbf{E} &= \frac{2R^2}{\sqrt{\Gamma} r_0^2} x_{min}^{3/2} \frac{1}{(1 + \frac{\Delta\theta_D^2}{x_*^2})^{1/2}} \\ \mathbf{l} &= \frac{\Delta\theta_D}{x_*} \frac{R}{(1 + \frac{\Delta\theta_D^2}{x_*^2})^{1/2}} . \end{aligned} \quad (51)$$

since

$$G = \int_1^{x_*/x_{min}} du \frac{u^{3/2}}{\sqrt{u^3 - 1}} , \quad (52)$$

and $G \sim x_*/x_{min}$ diverges linearly as $x_{min}/x_* << 1$.

Using the previous results *cf.* (30,51), we find that

$$H\mathbf{E}^2 \equiv \frac{\Gamma r_0^4 \mathbf{E}^2}{4R^4 x^3} < \frac{1}{1 + \frac{\Delta\theta_D^2}{x_*^2}} \equiv \frac{\mathbf{l}^2}{R^2} \left(\frac{\Delta\theta_D^2}{x_*^2} \right)^{-2} , \quad (53)$$

therefore \mathbf{l}^2/r^2 is much larger than $H\mathbf{E}^2$ for $r \geq R^*$ provided

$$\frac{\Delta\theta_D}{x_*} >> 1 . \quad (54)$$

Given $\Delta\theta_D$, which will ultimately define the quark separation distance along the deconstructed dimension, we thus choose the cut-off x_* in such a way as to verify the condition (54). This implies that the width of the deconstructed region we consider is sizeably smaller than the angle covered by the Wilson line in this deconstruction region.

5.2 Length and twist of the Wilson line outside the deconstruction domain

Since the quarks sources are initially placed at infinity, one cannot make definite conclusions on the interquark potential without studying the outside of the deconstruction domain. Let us thus discuss the geometrical features of the Wilson surface solution when $r > R^*$. Following the indications of Fig.3, the discussion may imply two regions, one with $r > \tilde{R}$ where one can use the conformal $AdS_5 \times S_5$ metrics and one transition region $R^* < r < \tilde{R}$.

The discussion depends mainly on the range of values of the twist $\Delta\theta_D$ one considers within the deconstruction domain. If we are considering $\Delta\theta_D$ large enough, which will correspond to a large distance in the deconstructed fifth dimension, then the relation (54) stands for finite values of x_* . This means that the transition region $R^* < R < \tilde{R}$ is small and thus not so much contributing to the potential.

Using then (53), one considers valid the condition

$$HE^2 < \frac{l^2}{r^2} . \quad (55)$$

This implies that the Lagrangian for $r > R_*$ becomes

$$\mathcal{L} = \frac{r}{(r^2 - l^2)^{1/2}} . \quad (56)$$

This is notably different from the case of non-rotating strings in the $AdS_5 \times S_5$ case, where the HE^2 term is the only non-trivial contribution in the Lagrangian. Notice that the previous approximation is valid in all the region outside the deconstructed domain. The contribution to the string length outside the deconstructed region is

$$L_F = 2 \frac{E\Gamma r_0^4}{l^3} \int_{R^*/l}^{\infty} \frac{u^2 + u_R^2}{(u^2 - u_R^2)^3} \frac{u}{(u^2 - 1)^{1/2}} du \quad (57)$$

where $u_R = R/l$. This is negligible for $x_{min} \ll 1$ as it scales as x_{min}^3 while L_D scales like $1/\sqrt{x_{min}}$. We then obtain that

$$L \approx L_D, L_F \ll L_D , \quad (58)$$

an approximation which improves for large L .

The situation is different for the bending of the string (see Fig.3). The bending angle of the string in the far-away region is given by

$$\Delta\theta_F = \int_{r^*/l}^{\infty} \frac{du}{u(u^2 - 1)^{1/2}} \equiv \pi/2 - \arctan \sqrt{\left(\frac{r^*}{l}\right)^2 - 1} \quad (59)$$

which is a finite quantity. For large values $\Delta\theta_D/x_* \gg 1$ we find that $l = R$ and $\Delta\theta_F \rightarrow \frac{\pi}{2}$. Notice that the bending in the far-away region saturates. Hence it is possible to ascribe a definite initial position to the source quarks in the far-away region in such a way that the minimal Wilson contour travels over the deconstruction region for a given value of $\Delta\theta_D$.

If we are now considering $\Delta\theta_D$ smaller, which will correspond to smaller distances in the deconstructed fifth dimension, then the relation (54) requires $x_* \ll 1$. This also means that the transition region $R^* < R < \tilde{R}$ is large and will require using the interpolating function H of (29) for the minimization. Without considering this more complex minimization problem, let us note that the function (29) contains singular terms in $1/x^2$ and $1/x$ which are expected to modify the interquark potential obtained from the $1/x^3$ term of (30). All in all, this means that our predictions are only valid for the large angular distance behaviour of the potential.

6 The static interquark potential: results and comments

We can now be more specific and evaluate the various contributions to the static potential for the large distance regime (large $\Delta\theta_D$).

The potential from the deconstruction region is simply

$$V_D = \frac{Rx_{min}}{2\pi\alpha'} \sqrt{G^2 x_{min}^2 + \Delta\theta_D^2} . \quad (60)$$

As L_D increases this gives

$$V_D = \frac{ng^2 F^2 R_5}{2\pi L_D^2} \sqrt{x_*^2 + \Delta\theta_D^2} . \quad (61)$$

The first term leads to the expected $1/L^2$ behaviour for a five-dimensional theory. When $\Delta\theta_D \gg x_*$ this reads

$$V_D = \frac{ng^2 F^2 x_5}{2\pi L_D^2} \quad (62)$$

where we have put $x_5 = R_5 \Delta\theta_D$. The potential is proportional to the 't Hooft coupling $g^2 n$ while the potential is in $(g^2 n)^{1/2}$ for the usual AdS case. Moreover V_D is both proportional to R_5 and $\Delta\theta_D$. It is tempting to interpret the combination $x_5 = R_5 \Delta\theta_D$ as a fifth dimensional distance between the quarks. In that case, the behaviour of the potential is reminiscent of confinement along the fifth dimension. It has to be noted that the would-be string tension

$$\mathcal{T} = \frac{g^2 n F^2}{2\pi L^2} \quad (63)$$

depends on the four-dimensional distance between the quarks, but this dependence is valid only in the limited region $L \ll (g^2 n)^{1/2} R_5$ (cf. (48)).

Let us briefly comment on the physical meaning of the potential $V_D(\Delta\theta_D, L_D)$. If we had inserted the probe D3 branes (carrying the Wilson loops) at the edge of the deconstructed region, i.e. at R^* , we would have had massive W -bosons in the theory and $V_D(\Delta\theta_D, L_D)$ would be related to their interaction potential. However in order to make the sources infinitely massive, and hence to have the analogues of quarks in the fundamental representation, we have to move the probe D3 branes and the Wilson loop away to $r \rightarrow \infty$. Then the θ parameter is modified by θ_F i.e. $\Delta\theta \rightarrow \Delta\theta_D + \Delta\theta_F$ according to (59). This drives the angles $\Delta\theta$ to larger values and changes the potential. Nevertheless the contribution to the potential from the far away region is nearly trivial as is shown by the following computation. Hence, the main contribution to the potential comes from the deconstructed region and can be interpreted as the interaction potential as seen by static quarks on the edge of the deconstructed domain.

For completeness, let us estimate the contribution to the potential from the domain far outside the deconstruction region. One obtains

$$\Delta V = V_F - V_0 \quad (64)$$

where V_F is the contribution for $r \geq R^*$ and V_0 is a regularization associated with an infinitely massive straight string. The contribution from the far-away region is

$$V_F - V_0 = \frac{l}{2\pi\alpha'} \int_{r_*/l}^{\infty} du \left[\frac{u}{(u^2 - 1)^{1/2}} - 1 \right] \equiv \frac{1}{2\pi\alpha'} \left(r^* - \sqrt{r^{*2} - l^2} \right) \quad (65)$$

Notice that this contribution is independent of L . We are particularly interested in the large $\Delta\theta_D \gg x_*$ region. In that case $\mathbf{l} = R$ and therefore this reduces to a constant contribution to the potential.

Let us end with some comments on our results. Having analysed non supersymmetric quiver theories in their deconstructed phase both at small and large coupling, we have retrieved the expected fifth dimensional behaviour. In particular, examining the angular dependence of the quark potential we have found a striking difference between the small and large coupling regimes. In the former, the force between the quarks decreases and vanishes for antipodal quarks. This implies that static quarks behave like non-interacting particles as long as they sit at antipodal points in the orbifold. At strong coupling we have found that the potential increases linearly with the angular distance, mimicking a confining behaviour along the large compact extra dimension.

These results are valid as long as $L \ll (g^2 n)^{1/2} R_5$. In the deep infrared regime where $L \gg (g^2 n)^{1/2} R_5$, the open string probes a region where $x \ll 1/\Gamma$. In this regime the harmonic function H becomes identical with the one of a single stack of n D3 branes. This corresponds to the fact that at very low energy the deconstructed quiver theory is a $N = 4$ $U(n)$ gauge theory. As well-known from deconstructed models, it is only in a finite energy range, i.e. a finite interval in L , that the gauge theory looks five-dimensional.

The situation would be entirely different if we had considered $R \leq \sqrt{\alpha'}$. In that case, the non-supersymmetry of the configuration is signaled by the presence of a twisted tachyon deforming the geometry of space-time. The analysis of the dual gauge theory in this case is beyond the scope of this paper[10].

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